Maximizing Economic Welfare in Euphemia

Analyzing Euphemia's objective function and its impact on consumer surplus, producer surplus, and congestion rent

Table of Contents

1 Introduction

Euphemia is the market-clearing solution used for SDAC and SIDC/IDA, and aims to maximize the economic welfare of market participants. Euphemia takes supply orders, demand orders, and network constraints as inputs and computes the acceptance of the orders, market prices for the different bidding zones and time periods, and the scheduled exchanges between the bidding zones.

The computation in Euphemia is split into multiple operational steps:
(1) Euphemia computes the acceptance of the orders by maximizing the total economic welfare in the so-called "primal problem". In this step, the market prices of the bidding zones remain undefined, as they are only computed in the following dual problem. While the primal problem computes the welfare, it does not determine how it is shared among market participants. Due to block orders and scalable complex orders, the resolution of this problem requires branching on binary variables to ensure that they take only values of zero or one.
(2) Euphemia computes the market prices of the different bidding zones in the dual problem. Market prices are restricted by the results of the primal problem to ensure that participation in the respective market is individually rational; that is, losses are avoided for each market participant. This implies that orders should have a non-negative surplus if they are accepted and simple orders that are rejected should have a non-positive surplus if they were accepted at the current market prices. However, these constraints do not always fix market prices fully due to indeterminacies. Note that the dual problem can be infeasible when some orders have been forced to be accepted due to branching. In case of such infeasibilities the respective primal solution has to be rejected and other primal solutions have to be considered.
(3) The dual problem is followed by additional sub-problems like the maximization of the accepted order volumes and other secondary objectives, which are performed without changing the total economic welfare or the prices.

The following text will explore in detail the objective of total economic welfare maximization, which is driving the order acceptance in the primal problem.

2 Welfare optimization in Euphemia

In this section, we provide definition and interpretation of the total economic welfare which is the primary objective maximized by Euphemia. It is defined as the sum of the producer surplus, consumer surplus, and congestion income. Here, consumer surplus is the valuation of the buyers for the received energy, represented by their bids, minus the required payment for this energy, producer surplus is the payment received by the sellers for the delivered energy minus the offered price , and congestion rent represents the economic value of the scarce transmission capacity. In particular, we assume here implicitly that the bid of a consumer corresponds to his monetary valuation for consuming the respective amount of energy and that the bid of a producer corresponds to his costs for generating the respective amount of energy. The congestion rent is defined by the product of the price spread and the scheduled exchange between two bidding zones.

For better interpretation of the congestion rent, one could consider the following thought experiment: Assume that a congested line is not directly modelled within Euphemia but that the owner of the line places a sell order in the importing area and a buy order in the exporting area. The volume of the sell order corresponds to the imported energy and the volume of the buy order corresponds to the exported energy. Consider the orders to be price taking to ensure them being fully accepted. In this case, the owner of the line would earn exactly the congestion income. The same principle can be extended to flow-based models where instead of lines causing congestion, the congestion is caused by network elements. This allows for more realistic and complex network definitions, as a single network element can be impacted by more than two bidding zones.

Total economic welfare can also be expressed in an alternative form which we will use in the context of optimization. Let us first consider the case of a single bidding zone. In this case, it is clear that congestion income has to be zero and that total economic welfare is just the sum of producer and consumer surplus. As the payment by the consumers corresponds to the payment received by the producers, it follows that total economic welfare is the valuation of the consumers for the consumed energy minus the costs for the producers to generate the energy. As there are no losses in the case of a single bidding zone consumed and produced energy matches. Interestingly the same condition still applies when considering multiple bidding zones: total economic welfare remains the valuation of the consumers for the consumed energy minus the cost for generating the required energy. In this case, the balance is guaranteed under the consideration of losses. This equivalent definition is used in Euphemia's primal problem. In other words, Euphemia's primal model does not contain any variable modelling market prices which would be needed to compute consumer surplus, producer surplus, and congestion income. Moreover, Euphemia never models congestion income, and only indirectly impacts it when optimizing the total economic welfare in the primal problem.

As we will see in the next sections, the objective of total economic welfare can be derived from basic market rules showing that it could not be replaced by an alternative objective without violating these rules at the same time.

3 Deriving the primal objective

In this section, we will introduce basic requirements from which the primal objective function that maximizes total economic welfare can be derived. We consider here a simplified version of the Euphemia model which considers only simple step orders. That means that we ignore block order, scalable complex orders, and interpolated orders. Moreover, we consider here only a single time period and ignore losses on lines. However, the principles also apply to the full model.

The basic requirements which we consider are as follows:

- No energy can be created, meaning that in any bidding zone, the production, consumption, and imports/exports sum up to zero.
- Orders must be accepted within their bounds.
- Orders can only be accepted if they would not lose money: accepted supply orders should not have order prices above the market prices and accepted demand orders should not have order prices below market prices.
- Orders can only be rejected if they would not win money: rejected supply orders should not have order prices below market prices and rejected demand orders should not have order prices above market prices.

Cross zonal interconnectors' that apply ATC model can only be used up to their capacity and can only earn congestion revenue if they are used at capacity.

- Flow-based constraints must be satisfied.
- Flow-based constraints can only cause congestion revenue if they are tight.

We show in Appendix 1 how these requirements can be used to derive the primal and dual model of Euphemia using the Karush-Kuhn-Tucker conditions (also known as the KKT conditions). These conditions must be valid for any optimal solution of an optimization problem that is fully derivable. Moreover, it holds that any solution respecting the KKT conditions must be optimal if the problem satisfies certain convexity criteria.

The primal model which we can derive from these basic requirements is presented below. The objective corresponds to the main objective of

Euphemia: the valuation for the accepted buyers minus costs of production for accepted suppliers.

Variables used in the model:

- v_o is the accepted volume of order o.
- V_o is the maximum acceptable volume of order o.
- P_o is the bid by order o.
- $f_{A,B}$ is the flow on the ATC line going from bidding zone A to bidding zone B. We assume here that either $f_{A,B}$ or $f_{B,A}$ is used for modelling the flows between two bidding zones and the direction is given by the sign.
- $F_{A,B}$ is the capacity of the ATC line from bidding zone A to bidding zone B in that direction.
- NP_A is the flow-based net position of bidding zone A. A positive net position corresponds to an exporting bidding zone.
- RAM_R is the remaining available margin of network element R.
- $PTDF_{R,A}$ is the contribution of the flow-based net position of bidding zone A on the load of network element R (including the balance of the full flow-based network as an additional constraint).
- B_A is the set of buying orders in bidding zone A.
- S_A is the set of selling orders in bidding zone A.

Primal model of Euphemia derived from basic requirements:

Maximise $\sum_{b \in B_A} P_b \; v_b - \sum_{s \in S_A} P_s v_s$

Such that:

$$\sum_{b \in B_A} v_b - \sum_{s \in S_A} v_s + \sum f_{A,*} - \sum f_{*,A} + NP_A = 0$$

$v_o \leq V_o$

$f_{A,B} \leq F_{A,B}$

$-f_{A,B} \leq F_{B,A}$

$\sum_A PTDF_{R,A} * NP_A \leq RAM_R$

$v_o \geq 0$

$f_{A,B}$ (no boundaries)

NP_A (no boundaries)

Since Euphemia's primal model can be derived from the basic requirements, any solution that satisfies these basic conditions must be optimal for Euphemia's model.

This means that there is no solution that would satisfy all the conditions above and does not have the same total welfare as the welfare being found by Euphemia's primal model.

In the next section, we will see what happens when binary orders and bidirectional flow avoidance are added to the model.

4 Non-convexities impacting welfare

The analysis above ignored the selection of block orders and scalable complex orders as those require binary variables which cannot be included in a linear program. Thus, these binary variables are relaxed during the problem resolution allowing any value between zero and one and become fixed to either zero or one when applying the branch and bound algorithm. The selection of such orders naturally can have an impact on the total economic welfare and the welfare distribution between consumers, producers and the network.

We have currently two sources of such non-convexities in Euphemia:
(1) Binary orders (fully/partially non-curtailable block orders and scalable complex orders) require branching during the computation, and are thus a source of non-convexity.

(2) Bi-directional flows need to be resolved. When prices are negative, losses can cause bidirectional flows on lines. This is caused by non-convexities of the losses which cannot be directly modelled in a linear model. As a consequence one of the directions needs to be chosen.

Both of these non-convexity sources cause the need for branching resulting in a tree structure in the search space. The only situation when Euphemia could select a solution with lower consumer and producer surplus and higher congestion income is when deciding between two different valid selections of blocks/flow-directions and when deciding to prune part of the search space due to sub-optimality with regards to total economic welfare.

5 Conclusion

In this document, we discussed the steps used to compute solutions in Euphemia. We focused in particular on the primal problem which determines the total economic welfare of a solution. Moreover, we showed how the objective function of Euphemia can be derived from fundamental requirements on the acceptance of orders. This showed that for each combination of binary values assigned to the binary variables of the problem, the respective total economic welfare is well defined.

References

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M. Madani and M. Starnberger, "Day-Ahead and Intraday Auctions Market Coupling Algorithm" in European Electricity Market Coupling, Springer Cham, 2025. Available: https://link.springer.com/book/10.1007/978-3-031-86315-8

Appendix 1 - Deriving Euphemia's objective function from basic constraints

In this appendix, we will derive the objective function used in Euphemia directly from basic requirements on the solutions which we will state below. The proof is based on the application of the Karush-Kuhn-Tucker (KKT) conditions.

Explaining the KKT conditions and their validity within Euphemia

We will express the KKT conditions in the context of a linear problem. While they are just as valid within a quadratic optimization problem, or even any convex problem with strong duality, the KKT conditions can be more easily reformulated and applied within linear problems. Thus, this annex will not consider interpolated orders which could cause quadratic terms in the objective function. It should be noted that the same conclusions still apply to cases with interpolated orders.

For the KKT conditions in the context of a linear problem, consider a primal problem and its corresponding dual:

max $c^T x$	min $b^T \mu$
s.t.:	s.t.:
$Ax \leq b$	$A^T \mu \geq c$
$x \geq 0$	$\mu \geq 0$

Then the following applies for each optimal solution: (1) For each primal constraint, either the constraint is binding, or the corresponding dual variable is at its bound. (2) For each primal variable, either it is at its bound, or the corresponding dual constraint is binding.
Moreover, any valid point of both the primal and the dual satisfying the above conditions is a global optimum of the optimization problem.

We will use this theory to show that we can consider basic market requirements, defined in the following sections, and derive from them the objective function of Euphemia, as well as, both the primal and the dual models to be used.

Limitations of the KKT Conditions

There are a few limitations to this analysis:

- Mixed-integer problems (MIPs) such as Euphemia's primal problem contain both continuous and integer (binary in case of Euphemia) variables. Within such problems the KKT conditions are only valid for local optima referring to a single assignment to the integer/binary variables. However, KKT conditions still provide global optima for all problems containing relaxed integer/binary variables, which are solved sequentially by Euphemia during the resolution of the primal problem.
- Euphemia solves the market optimization problem in multiple successive steps. The objective function of Euphemia only concerns the first step, determining the total economic welfare of a solution, and not following steps including price determination, volume maximization, and others.

Basic requirements of Euphemia

Here are the most basic requirements of Euphemia which we will consider below:

- No energy can be created, meaning that in any bidding zone, the production, consumption, and imports/exports sum up to zero.. This is what will be called the *balance constraint*.
- Orders must be accepted within their bounds.
- Orders can only be accepted if they would not lose money (i.e., no paradoxical acceptance).
- Orders can only be rejected if they would not win money (i.e., no paradoxical rejection).
- ATC lines can only be used up to their capacity.
- ATC lines can only earn congestion revenue if they are used at capacity.
- Flow-based constraints must be respected.
- Flow-based constraints can only cause congestion revenue if they are tight; that is, which means that the RAM of the respective CNEC is fully used.

Note that the requirements above on orders being accepted or rejected can be impacted by branching for orders based on binary variables (e.g., block orders). Such orders are thus ignored in this study.[1]

These requirements can be expressed in mathematical terms with the following convention:

- v_o will be the accepted volume of order o.
- V_o will be the maximum acceptable volume of order o, and cannot be 0.
- The minimum acceptable volume of any order is 0.
- P_o is the bid by order o.
- π_A is the price in bidding zone A.
- $f_{A,B}$ is the flow on the ATC line going from bidding zone A to bidding zone B.
- $f_{B,A}$ is the flow on the ATC line going from bidding zone B to bidding zone A. We assume here that either $f_{A,B}$ or $f_{B,A}$ is used for modelling the flows between two bidding zones and the direction is given by the sign.
- $F_{A,B}$ is the capacity of the ATC line from bidding zone A to bidding zone B in that direction.
- $\rho_{A,B}$ is the congestion income per MW of usage of the ATC line going from bidding zone A to bidding zone B, and is equal to the difference in prices between bidding zone A and bidding zone B if positive or zero otherwise. $\rho_{B,A}$ will be used in its place when considering flows from B to A.
- NP_A is the flow-based net position of bidding zone A, which considers only flows within the flow-based area. A positive net position corresponds to an exporting bidding zone.
- RAM_R is the remaining available margin of network element R.
- $PTDF_{R,A}$ is the contribution of the flow-based net position of bidding zone A on the load of network element R (including the balance of the full flow-based network as an additional constraint).
- ρ_R is the congestion income per MW of usage of network constraint R.

[1] We here do not consider orders that are branched on. This is due to the fact that if the lower bound on the acceptance of an order is binding and that order has been branched on, the price problem will not be valid, and the solution at that branch will have to be rejected. As we are here looking at one single branch, investigating non-acceptable branches goes beyond the scope of this annex.

- B_A is the set of buying orders in bidding zone A.
- S_A is the set of selling orders in bidding zone A.
- Subscript $*$ denotes all possible values of that subscript.

We then have the following expression of the basic constraints:

1. $\sum_{s \in S_A} v_s - \sum_{b \in B_A} v_b = NP_A + \sum f_{A,*} - \sum f_{*,A}$ or equivalently$\sum_{s \in S_A} v_s - \sum_{b \in B_A} v_b - NP_A - \sum f_{A,*} + \sum f_{*,A} = 0$ (The balance constraint)
2. $0 \leq v_o \leq V_o$ (Orders must be accepted within bounds)
3. Not both $0 < v_b$ and $\pi_A > P_b$ for any order $b \in B_A$(Orders cannot be accepted and lose money) (equivalently with orthogonality notation: $0 \leq v_b \perp \pi_A \geq P_b$)
4. Not both $0 < v_s$ and $\pi_A < P_s$ for any order $s \in S_A$(Orders cannot be accepted and lose money) (equivalently with orthogonality notation: $0 \leq v_s \perp \pi_A \leq P_s$)
5. Not both $v_b < V_b$ and $\pi_A < P_b$ for any order $b \in B_A$(Orders cannot be partially or fully rejected and would win money) (equivalently with orthogonality notation: $v_b \leq V_b \perp \pi_A \leq P_b$)
6. Not both $v_s < V_s$ and $\pi_A > P_s$ for any order $s \in S_A$(Orders cannot be partially or fully rejected and would win money) (equivalently with orthogonality notation: $v_s \leq V_s \perp \pi_A \geq P_s$)
7. $f_{A,B} \leq F_{A,B}$ (Lines cannot be used more than their capacity)
8. Not both $f_{A,B} < F_{A,B}$ and $\rho_{A,B} > 0$ (Lines cannot win money while not being used fully) (equivalently with orthogonality notation: $f_{A,B} \leq F_{A,B} \perp \rho_{A,B} \geq 0$)
9. $\sum NP_* * PTDF_{R,*} \leq RAM_R$ (Flow-based constraints need to be respected; we include here two mandatory flow-based constraints which enforce that the sum of the net-positions equals zero: one constraint with all coefficients being 1 and one constraint with all coefficient being -1; the RAM being zero in both cases)
10. Not both $\sum NP_* * PTDF_{R,*} < RAM_R$ and $\rho_R > 0$ (equivalently with orthogonality notations: $\sum NP_* * PTDF_{R,*} \leq RAM_R \perp \rho_R \geq 0$)

Going from the basic requirements to Euphemia's primal model

Let us start by setting π_A as the dual variable for bidding zone A's balance constraint.

We can then focus on Constraints 3 and 5:

- First of all, we know that we will have $0 \leq v_b$ and $v_b \leq V_b$ as primal constraint, per Constraint 2. We introduce a new dual variable σ_b as dual variable of $v_b \leq V_b$.
- We can now express the dual parts of Constraint 3 and 5 by $\pi_A + \sigma_b \geq P_b$, with this dual constraint being linked to variable v_b.
- The KKT conditions require that not both $\pi_A + \sigma_b > P_b$ and $0 < v_b$ are satisfied. If $0 = v_b$, then by definition $v_b < V_b$, which implies that $\sigma_b = 0$ and that $\pi_A + \sigma_b > P_b$ equals $\pi_A > P_b$. Otherwise, if $0 < v_b$ then $\sigma_b \geq 0$ and $\pi_A + \sigma_b = P_b$. This means $\pi_A \leq P_b$. Thus, not both conditions $\pi_A > P_b$ and $0 < v_b$ are satisfied corresponding to Constraint 3.
- The KKT condition on $v_b \leq V_b$ is that not both conditions $v_b < V_b$and $\sigma_b > 0$ are satisfied. If $v_b < V_b$, then $\sigma_b = 0$ and $\pi_A + \sigma_b \geq P_b$ equals $\pi_A \geq P_b$. It follows that not both conditions $v_b < V_b$ and $\pi_A < P_b$ are satisfied corresponding to Constraint 5.

Similar techniques can be applied for Constraints 4 and 6 to receive the dual constraint $\pi_A - \sigma_s \leq P_s$.

Constraints 8 and 10 represent regular KKT conditions.

We now have a set of dual constraints and variables:

- $\pi_A + \sigma_b \geq P_b$ (v_b)
- $-\pi_A + \sigma_s \geq -P_s$ (v_s)
- $\rho_{A,B} - \rho_{B,A} = \pi_B - \pi_A$ $(f_{A,B})$
- $\sum_R \quad PTDF_{R,A} * \rho_R - \pi_A = 0$ (NP_A)[2]
- π_A (no boundaries)
- $\sigma_o \geq 0$
- $\rho_{A,B} \geq 0$
- $\rho_R \geq 0$

This allows us to derive a primal problem *with objective*.

Maximise $\sum_b \quad P_b \quad v_b - \sum_s \quad P_s v_s$

Such that:

$\sum_{b \in B_A} \quad v_b - \sum_{s \in S_A} \quad v_s + \sum \quad f_{A,*} - \sum \quad f_{*,A} + NP_A = ?$ (π_A)

$v_o \leq ?$ (σ_o)

$f_{A,B} \leq ?$ $(\rho_{A,B})$

[2] The constraint has been derived by dualising constraints 1 and 9.

$-f_{A,B} \leq ?$ $(\rho_{B,A})$

$\sum_A \quad PTDF_{A,R} * NP_A \leq ?$ (ρ_R)

$v_o \geq 0$

$f_{A,B}$ (no boundaries)

NP_A (no boundaries)

We can then complete the right hand side of our inequalities to match our basic Euphemia requirements:

Maximise $\sum_b \quad P_b \quad v_b - \sum_s \quad P_s v_s$

Such that:

$\sum_{b\in B_A} \quad v_b - \sum_{s\in S_A} \quad v_s + \sum \quad f_{A,*} - \sum \quad f_{*,A} + NP_A = 0$ (π_A)

$v_o \leq V_o$ (σ_o)

$f_{A,B} \leq F_{A,B}$ $(\rho_{A,B})$

$-f_{A,B} \leq F_{B,A}$ $(\rho_{B,A})$

$\sum_A \quad PTDF_{A,R} * NP_A \leq RAM_R$ (ρ_R)

$v_o \geq 0$

$f_{A,B}$ (no boundaries)

NP_A (no boundaries)

This allows us to find the objective of our dual problem:

Minimise $\sum_o \quad \sigma_o V_o + \sum_{A,B} \quad \rho_{A,B} F_{A,B} + \sum_R \quad \rho_R RAM_R$

It should be noted that the dual works in the infeasible part of the primal. While this might look like minimizing the surplus of all actors, it is actually doing so over part of the infeasible domain, and reaching the same value of the maximum surplus at any optimal solution.

Implications of deriving Euphemia from basic requirements

Since Euphemia's primal model can be entirely derived from the basic conditions written above, any solution that satisfies these basic conditions must be optimal for Euphemia's model.

This means that there is no solution[3] that would satisfy all the conditions above and does not have the same total welfare as the welfare being found (and fixed) by Euphemia's primal model.

[3] Due to the presence of multiple nodes, this is valid for any block selection, but not over various block selections.

Consequently, any modification in Euphemia's objective function corresponds to a change in the basic requirements.